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On the Mass Dependence of Diffusion within Biological Membranes and Polymers

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Summary. A simple probability argument suggests that the diffusion coefficient for biological membranes (and polymers) should vary exponentially with the reciprocal of the molecular weight of the diffusant. A test of this relationship shows that it fits the experimental data at least as well as the empirical relation previously proposed. Since the present treatment has some theoretical justification, it would seem to be preferred.

We wish to show how by a simple treatment we can derive an expression which describes the mass dependence of the diffusion coefficients in biological membranes.

Lieb and Stein [4] claimed that biological membranes behave as nonporous polymeric sheets with respect to the diffusion of nonelectrolytes. They proposed the relation

$$D_m = AM^{-S_m} \tag{1}$$

in which D_m is the diffusion coefficient of a nonelectrolyte with mass M in a membrane or polymer, and A and S_m are constants not dependent on the mass of the diffusant. Eq. (1) is approximately obeyed for diffusion in both biological membranes and polymers for some range of M values ($M_{\max}/M_{\min} \leqq 5$). However, an explanation as to why Eq. (1) should be valid and an interpretation of the quantity S_m were missing, and it has not proved possible to remedy this deficiency. We now show how one can replace Eq. (1) by another relation which can be simply derived and interpreted.

Let p be the probability that at any point in a homogeneous polymer or membrane there is a hole of volume v_0. Under the assumption that holes behave independently of each other (admittedly, a crude approximation), the probability of finding k holes in the region of the same point is p^k. Let $n(v)$ be the number of holes of volume $v(=kv_0)$ per unit volume of polymer or membrane. We set

$$n(v)=A_1\,p^k. \tag{2}$$

A_1 is a constant of proportionality. Let v^* be the critical volume for an adjacent hole required for the diffusion of a given substance. Let n be the number of holes (per unit volume) of volume greater than v^*.

$$\begin{aligned} n&=\int_{v^*}^{\infty} n(v)\,dv=A_1\int_{v=k^*v_0}^{\infty} p^k\,dv=A_1 v_0\int_{k^*}^{\infty} p^k\,dk=-\frac{v_0A_1}{\log p}\exp(k^*\log p)\\ &=\frac{v_0A_1}{|\log p|}\exp\left(-\frac{v^*}{v_0}\,|\log p|\right). \end{aligned} \tag{3}$$

Since D_m is proportional to n, the number of holes of volume greater than v^*, we set

$$D_m=A\exp(-v^*\,|\log p|/v_0) \tag{4}$$

in which v^* depends on the diffusant and p (and v_0) depend on the membrane or the polymer. A is regarded as being constant but probably varies although very slowly in relation to the exponential factor. The term v^* may also depend on the medium but we neglect such a dependence, the consideration of which would force one to enter into details of the mutual interaction. Eq. (4) is of the same form as Eq. (5) below, derived by Cohen and Turnbull [2]:

$$D=A\exp-\gamma\frac{v^*}{v_f}=A\exp-\frac{\gamma v^*}{\alpha\bar{v}_m(T-T_0)} \tag{5}$$

in which α and $\bar{v}_m$ are the mean values of the coefficient of thermal expansion and the molecular volume, respectively; γ is a numerical factor which has a value between $\frac{1}{2}$ and 1; T is the absolute temperature; and T_0 is the temperature at which the free volume v_f disappears. Actually A is not constant and has the same meaning as in Eq. (4). (*See* Ref. [2].)

In order to compare the prediction of Eq. (4) [or (5)] with experiments for both membranes and polymers, we further approximate by assuming that v^* is proportional to the molecular weight M, namely,

$$v^*=\beta M. \tag{6}$$

Substituting Eq. (6) into Eq. (4) and taking logarithms, we obtain:

$$\log D_m=B-\frac{\beta\,|\log p|}{v_0}M. \tag{7}$$

Table. *Least-squares analysis of the relation between log D and M and between log D and log M*

System	log D vs. log M		log D vs. M		Ref.
	Slope	σ[a]	Slope	σ[a]	
Chara[b]	−3.5	0.16	−0.016	0.18	[4]
Nitella[b, c]	−2.2	0.31	−0.008	0.35	[4]
Natural rubber, 1.7% sulphur, 40 °C	−1.21	0.06	−0.0151	0.10	[1][d]
„ „ 1.7% „ 60 °C	−1.18	0.04	−0.0150	0.07	[1][d]
„ „ 1.7% „ 80 °C	−1.05	0.04	−0.0134	0.06	[1][d]
„ „ 2.9% „ 40 °C	−1.22	0.07	−0.0161	0.07	[1][d]
„ „ 2.9% „ 60 °C	−1.04	0.04	−0.0135	0.05	[1][d]
„ „ 2.9% „ 80 °C	−0.91	0.03	−0.0117	0.04	[1][d]
„ „ 7.15% „ 40 °C	−1.84	0.03	−0.0238	0.06	[1][d]
„ „ 7.15% „ 60 °C	−1.64	0.06	−0.0216	0.06	[1][d]
„ „ 7.15% „ 80 °C	−1.43	0.06	−0.0185	0.06	[1][d]
„ „ 11.3% „ 40 °C	−2.42	0.10	−0.0249	0.13	[1][d]
„ „ 11.3% „ 60 °C	−1.89	0.06	−0.0243	0.09	[1][d]
„ „ 11.3% „ 80 °C	−1.65	0.04	−0.0211	0.08	[1][d]

[a] $\sigma = \sqrt{\sum_{i=1}^{n} (y_i - y_{ci})^2/(n-2)}$ where y_i stands for $\log D_i$ and y_{ci} is the value of $\log D_i$ given by either the straight line of log D vs. log M or of log D vs. M. n is the number of experimental points.

[b] Trimethylcitrate was excluded.

[c] Ethanol, n-propanol, tert-butanol, antipyrene, iso-valeramide, glycerol monomethyl ether and glycerol monoethyl ether taken from Ref. [3] are included in addition to the substances in Ref. [4].

[d] The diffusants are methane, ethane, ethylene, propane and butane.

Eq. (7) predicts a linear relation between $\log D_m$ and M, whereas in Eq. (1) we have a linear relation between log D and log M. For a sufficiently narrow range of values for M, the logarithmic and linear relations will be difficult to distinguish, since $\log(M_1 + X) = \log M_1 + \frac{X}{M_1}$ to a sufficiently good approximation, for $\frac{|X|}{M_1} < \frac{1}{2}$.

A least-squares analysis of log D vs. M using the data of Ref. [4] gave an agreement with experiment about as fair as that achieved in Ref. [4] (Table). From Eq. (7) it is evident that the slope of the linear plot of log D vs. M should decrease in absolute magnitude when the probability of finding a hole of volume v_0 increases towards unity. This should occur when the polymer or membrane becomes less rigid, or more liquid-like. This was indeed observed when rubbers with different degrees of cross-linking were considered. (The data used is given in Ref. [1].) By the same argument, an increase of temperature should also increase p, as is seen more explicitly in Eq. (5). In our calculations, using the data in Ref. [1], the absolute magnitude of the slope of the plot of $\log D$ vs. M decreased by about 20% when the temperature rose from 40 to 80 °C (i.e., from about 310 to 350 °K). It should be pointed out that Eqs. (4)–(7) are very approximate in their nature and do not help us to distinguish between two diffusants according to their chemical properties or their conformation, the mass being the only property taken into account. Deeper consideration, however, suggests that the biggest non-spherical diffusants in an homologous series would be expected to have D values higher than those predicted by Eq. (7). This is because the rate of diffusion will be increased owing to the diffusing molecule arranging itself with its smallest cross section in the direction of its motion. Thus, v^* will not increase according to Eq. (6) but more slowly, resulting in a slower decrease of D_m with M than that predicted by Eq. (7).

We conclude, therefore, that a linear dependence of the logarithm of the diffusion coefficient on the molecular weight of the diffusant fits the experimental data at least as well as does the logarithmic/logarithmic expression previously used [4]. The logarithmic/linear form has an obvious theoretical justification, however, and should therefore be used in preference.

References

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